338. Proposed by RICHARD LOCHNER, Philadelphia, Pa.

An elliptical field has a major axis of 100 feet and a minor axis of 10 feet. A cow is tethered at the end of the major axis and another at the end of the minor axis. If each cow can graze over half the field, how long is the rope of each? What is the area of the portion over which the cows can graze in common?

MECHANICS.

274. Proposed by G. B. M. ZERR.

A sphere moves on the concave side of a rough cylindrical surface of which the transverse section perpendicular to the generating lines is a hypocycloid. If $s = l \sin n\theta$ be the intrinsic equation of the hypocycloid, then $l = (a - b)4b/a$, $n = a/(a - 2b)$, where a = radius of fixed circle, b = radius of rolling circle.

275. W. J. GREENSTREET, Editor Mathematical Gazette, England.

If a particle be attracted towards the angular points of a regular hexagon by forces equal to r^{-h}, at distance r, find the condition for stability of equilibrium.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

190. Proposed by H. C. FEEMSTER, York, Neb.

Show that, if n is a prime number and r is an integer less than n, then

$$\underline{|r-1}\ \underline{|n-r} + (-1)^{r-1} = M\cdot n,$$

where M is an integer.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

376. Proposed by W. W. BEMAN, Ann Arbor, Mich.

If

$$\left(1+\frac{1}{m}\right)^m/e = 1 - a_1\frac{1}{m} + a_2\frac{1}{m^2} - a_3\frac{1}{m^3} + \cdots,$$

prove that

$$na_n = \sum_{k=1}^{k=n} \frac{k}{k+1} a_{n-k}$$

and compute $a_1, a_2, a_3, \cdots, a_8$.

Solution by the Proposer.

For convenience, put $1/m = x$, and $y = (1+x)^{1/x}/e$. Then

$$ey = (1+x)^{1/x},$$

$$\log y = -\frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots$$

and

$$\frac{y'}{y} = -\frac{1}{2} + \frac{2x}{3} - \frac{3x^2}{4} - \cdots.$$

Assume

$$y = 1 - a_1x + a_2x^2 - a_3x^3 + \cdots.$$

Then

$$y' = -a_1 + 2a_2x - 3a_3x^2 + 4a_4x^3 - \cdots$$
$$= \left(1 - a_1x + a_2x^2 - a_3x^3 + \cdots\right)\left(-\frac{1}{2} + \frac{2x}{3} - \frac{3x^2}{4} + \cdots\right).$$

Equating coefficients,

$$a_1 = \tfrac{1}{2}, \qquad 2a_2 = \tfrac{1}{2}a_1 + \tfrac{2}{3},$$

$$3a_3 = \tfrac{1}{2}a_2 + \tfrac{2}{3}a_1 + \tfrac{3}{4},$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$na_n = \tfrac{1}{2}a_{n-1} + \tfrac{2}{3}a_{n-2} + \tfrac{3}{4}a_{n-3} + \cdots + \frac{n}{n+1}$$

$$= \sum_{k=1}^{k=n} \frac{k}{k+1} a_{n-k}.$$

The first six values of a_n are

$$a_1 = \tfrac{1}{2}, \quad a_2 = \frac{11}{24}, \quad a_3 = \frac{7}{16}, \quad a_4 = \frac{2447}{5760}, \quad a_5 = \frac{459}{2304}, \quad a_6 = \frac{238043}{580608}.$$

See solution of 377 in the December issue.

Solutions of 377 and 378 were received, too late for credit in the December issue, from C. N. Schmall and H. E. Trefethen; also of 378 from A. L. McCarty.

GEOMETRY.

409. Proposed by S. LEFSCHETZ, University of Nebraska.

The sides of a triangle being in arithmetic progression, a and a' being, respectively, the smallest and largest sides, r, R the radii of the inscribed and circumscribed circles, to prove that $6Rr = aa'$.

SOLUTION BY W. C. EELLS, Tacoma, Wash.

Since the sides are in arithmetic progression, represent them by

$$a, \quad \frac{a+a'}{2}, \quad a'.$$

Then from trigonometry, we have

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}, \quad \text{and} \quad R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where s is the half-sum of the three sides a, b, c.

Substituting $s = \dfrac{3(a+a')}{4}$ and reducing, we have

$$r = \frac{1}{4}\sqrt{\frac{(3a'-a)(3a-a')}{3}}, \quad \text{and} \quad R = \frac{2aa'}{\sqrt{3(3a'-a)(3a-a')}}$$

whence $rR = \dfrac{aa'}{6}$ and $6rR = aa'$.

Also solved by J. Scheffer, C. N. Schmall, Levi S. Shively, A. M. Harding, and A. L. McCarty.